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Criteria for the first order Fréedericksz transitions

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This paper summarizes the exact criteria for the first order Fréedericksz transition to occur in magnetically, electrically or optically driven liquid crystal cells. All these criteria are in terms of the material parameters, cell geometrical parameters and surface anchoring parameters.

1. Introduction

The Fréedericksz transition has been known for over 70 years. Recently, the exact criteria for the first order transition driven by a magnetic field [1] and an optical field [2] were developed. This paper puts these results, as well as the criteria for the electrical case, together and gives some brief remarks.

2. Criteria for the first order Fréedericksz transition

Consider four types of liquid crystal cell: the electrically driven planar cell (EP), the magnetically driven planar cell (MP), the magnetically driven homeotropic cell (MH) and the optically driven homeotropic cell (OH)—see figure 1. θ is the angle between the easy direction and the director, and θ_0, θ_m are evaluated at the surface and the middle layer. The applied magnetic field, electrical voltage and incident light intensity are denoted by H, V and I, respectively. $\Delta\chi$, $\Delta\varepsilon$ are the magnetic and electrical anisotropy defined conventionally. Subscripts $\parallel$ and $\perp$ refer to the directions along and perpendicular to the local director, n_o, n_e are the principle values of the refractive index ellipse, k_{11}, k_{22}, k_{33} are the elastic constants. S and d are the substrate area and cell thickness. Both substrates are assumed to be physically identical. The anchoring strengths at the surfaces are described by the Rapini surface anchoring energy

$$g_s = \frac{1}{2}(A_2 \sin^2 \theta_0 + A_4 \sin^4 \theta_0) \qquad (1)$$

where A_2 and A_4 are anchoring coefficients. The $\hat{z}$ axis is set to be along the normal to the substrate; therefore $\theta = \theta(z)$. Symmetry about the middle layer exists and, thus θ_m is the largest deformation angle and can be chosen as the order parameter to describe the deformation. For

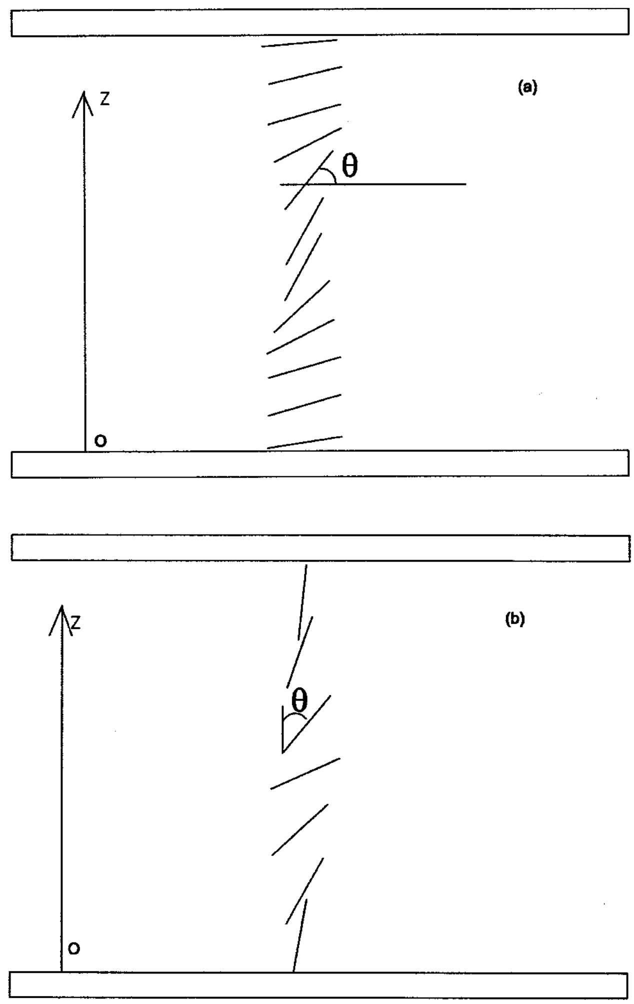

Figure 1. Planar cell (*a*) and homeotropic cell (*b*).

the sake of convenience, cgs units are used for the magnetically induced and optically induced transitions and SI units for the electrical case. Under the action of

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the external field, the free energy of the system is:

$$F = \begin{cases} S\int_0^d \left[\frac{1}{2}(k_{11}\cos^2\theta + k_{33}\sin^2\theta)\left(\frac{d\theta}{dz}\right)^2\right]dz + S(A_2\sin^2\theta_0 + A_4\sin^4\theta_0) - \frac{1}{2}\Delta\chi H^2 S\int_0^d \sin^2\theta\, dz & \text{for MP,} \\ S\int_0^d \left[\frac{1}{2}(k_{11}\cos^2\theta + k_{33}\sin^2\theta)\left(\frac{d\theta}{dz}\right)^2\right]dz + S(A_2\sin^2\theta_0 + A_4\sin^4\theta_0) - \frac{1}{2}\varepsilon_0\Delta\varepsilon V^2 \frac{S}{\int_0^d \frac{dz}{(\varepsilon_\perp\cos^2\theta + \varepsilon_\parallel\sin^2\theta)}} & \text{for EP,} \\ S\int_0^d \left[\frac{1}{2}(k_{11}\sin^2\theta + k_{33}\cos^2\theta)\left(\frac{d\theta}{dz}\right)^2\right]dz + S(A_2\sin^2\theta_0 + A_4\sin^4\theta_0) - \frac{1}{2}\Delta\chi H^2 S\int_0^d \sin^2\theta\, dz & \text{for MH,} \\ S\int_0^d \left[\frac{1}{2}(k_{11}\sin^2\theta + k_{33}\cos^2\theta)\left(\frac{d\theta}{dz}\right)^2\right]dz + S(A_2\sin^2\theta_0 + A_4\sin^4\theta_0) - \frac{n_o n_e I}{c} S\int_0^d \frac{dz}{(n_e^2\cos^2\theta + n_o^2\sin^2\theta)^{1/2}} & \text{for OH.} \end{cases} \tag{2}$$

The non-trivial solutions to the Euler–Lagrange equation are found to be:

$$z(\theta) = \begin{cases} \frac{1}{H(\Delta\chi)^{1/2}}\int_{\theta_0}^{\theta}\left(\frac{k_{11}\cos^2\theta + k_{33}\sin^2\theta}{\sin^2\theta_m - \sin^2\theta}\right)^{1/2} d\theta & \text{for MP,} \\ \frac{(\varepsilon_\perp\cos^2\theta_m + \varepsilon_\parallel\sin^2\theta_m)^{1/2}}{V(\varepsilon_0\Delta\varepsilon)^{1/2}}\int_0^d \frac{dz}{(\varepsilon_\perp\cos^2\theta + \varepsilon_\parallel\sin^2\theta)} \times \int_{\theta_0}^{\theta}\left[\frac{(k_{11}\cos^2\theta + k_{33}\sin^2\theta)(\varepsilon_\perp + \Delta\varepsilon\sin^2\theta)}{\sin^2\theta_m - \sin^2\theta}\right]^{1/2} d\theta & \text{for EP,} \\ \frac{1}{H(\Delta\chi)^{1/2}}\int_{\theta_0}^{\theta}\left(\frac{k_{11}\sin^2\theta + k_{33}\cos^2\theta}{\sin^2\theta_m - \sin^2\theta}\right)^{1/2} d\theta & \text{for MH,} \\ \left(\frac{c}{2In_o n_e}\right)^{1/2}(n_e^2\cos^2\theta_m + n_o^2\sin^2\theta_m)^{1/4} \times \int_{\theta_0}^{\theta}\left[\frac{(k_{11}\sin^2\theta + k_{33}\cos^2\theta) \times (n_e^2\cos^2\theta + n_o^2\sin^2\theta)^{1/2}}{(n_e^2\cos^2\theta + n_o^2\sin^2\theta)^{1/2} - (n_e^2\cos^2\theta_m + n_o^2\sin^2\theta_m)^{1/2}}\right]^{1/2} d\theta & \text{for OH.} \end{cases} \tag{3}$$

The field can be expressed in terms of θ_0, θ_m:

$$T = \begin{cases} H = \frac{2}{d(\Delta\chi)^{1/2}}\int_{\theta_0}^{\theta_m}\left(\frac{k_{11}\cos^2\theta + k_{33}\sin^2\theta}{\sin^2\theta_m - \sin^2\theta}\right)^{1/2} d\theta & \text{for MP,} \\ V = \frac{2}{(\varepsilon_0\Delta\varepsilon)^{1/2}}(\varepsilon_\perp\cos^2\theta_m + \Delta\varepsilon\sin^2\theta_m)^{1/2} \times \int_{\theta_0}^{\theta_m}\left[\frac{k_{11}\cos^2\theta + k_{33}\sin^2\theta}{(\varepsilon_\perp\cos^2\theta + \varepsilon_\parallel\sin^2\theta)(\sin^2\theta_m - \sin^2\theta)}\right]^{1/2} d\theta & \text{for EP,} \\ H = \frac{2}{d(\Delta\chi)^{1/2}}\int_{\theta_0}^{\theta_m}\left(\frac{k_{11}\sin^2\theta + k_{33}\cos^2\theta}{\sin^2\theta_m - \sin^2\theta}\right)^{1/2} d\theta & \text{for MH,} \\ I = \frac{2c}{n_o n_e d^2}(n_e^2\cos^2\theta_m + n_o^2\sin^2\theta_m)^{1/2} \times \left\{\int_{\theta_0}^{\theta_m}\left[\frac{(k_{11}\sin^2\theta + k_{33}\cos^2\theta) \times (n_e^2\cos^2\theta + n_o^2\sin^2\theta)^{1/2}}{(n_e^2\cos^2\theta + n_o^2\sin^2\theta)^{1/2} - (n_e^2\cos^2\theta_m + n_o^2\sin^2\theta_m)^{1/2}}\right]^{1/2} d\theta\right\}^2 & \text{for OH.} \end{cases} \tag{4}$$

Boundary conditions:

$$\int_{\theta_0}^{\theta_m}\left(\frac{k_{11}\cos^2\theta + k_{33}\sin^2\theta}{\sin^2\theta_m - \sin^2\theta}\right)^{1/2} d\theta = \frac{d}{2}\frac{\sin\theta_0\cos\theta_0}{[(k_{11}\cos^2\theta_0 + k_{33}\sin^2\theta_0)(\sin^2\theta_m - \sin^2\theta_0)]^{1/2}} \times (A_2 + 2A_4\sin^2\theta_0) \quad \text{for MP,}$$

$$\int_{\theta_0}^{\theta_m}\left[\frac{(k_{11}\cos^2\theta + k_{33}\sin^2\theta)(\varepsilon_\perp\cos^2\theta + \varepsilon_\parallel\sin^2\theta)}{\sin^2\theta_m - \sin^2\theta}\right]^{1/2} d\theta = \frac{d}{2}\left[\frac{(\varepsilon_\perp\cos^2\theta_0 + \varepsilon_\parallel\sin^2\theta_0)}{(k_{11}\cos^2\theta_0 + k_{33}\sin^2\theta_0)(\sin^2\theta_m - \sin^2\theta_0)}\right]^{1/2} \times \sin\theta_0\cos\theta_0(A_2 + 2A_4\sin^2\theta_0) \quad \text{for EP,}$$

Table 1. Criteria for the first order Fréedericksz transition. The definitions of the special functions f_{M1}, f_{M2}, f_{E1}, f_{E2}, f_{O1}, f_{O2} are listed in the appendix.

Cell type	Threshold criteria	Saturation criteria
MP	$\frac{k_{33}}{k_{11}} < -\frac{A_4}{A_2} f_{M1}(A_2 d/2k_{11})$	$\frac{k_{11}}{k_{33}} < -\frac{A_4}{A_2+2A_4} f_{M2}[(A_2+2A_4)d/2k_{33}]$
EP	$\frac{\varepsilon_\parallel}{\varepsilon_\perp} < \left(1-\frac{k_{33}}{k_{11}}\right) - \frac{k_{33}}{k_{11}} f_{E1}(A_2 d/2k_{11}) - \frac{A_4}{A_2} f_{E2}(A_2 d/2k_{11})$	$\frac{\varepsilon_\perp}{\varepsilon_\parallel} > 1 + \frac{k_{11}}{k_{33}} f_{E3}[(A_2+2A_4)d/2k_{33}] + \frac{A_4}{A_2+2A_4} f_{E4}[(A_2+2A_4)d/2k_{33}]$
MH	$\frac{k_{11}}{k_{33}} < -\frac{A_4}{A_2} f_{M1}(A_2 d/2k_{33})$	$\frac{k_{33}}{k_{11}} < -\frac{A_4}{A_2+2A_4} f_{M2}[(A_2+2A_4)d/2k_{11}]$
OH	$\frac{n_o^2}{n_e^2} < \left(1-\frac{4k_{11}}{9k_{33}}\right) + \frac{4k_{11}}{9k_{33}} f_{O1}(A_2 d/2k_{33}) - \frac{64}{3}\frac{A_4}{A_2} f_{O2}(A_2 d/2k_{33})$	$\frac{n_e^2}{n_o^2} > \left(1+\frac{4}{3}\frac{k_{33}}{k_{11}}\right) - \frac{4}{3}\frac{k_{33}}{k_{11}} f_{O3}[(A_2+2A_4)d/2k_{11}] + \frac{32A_4}{3(A_2+2A_4)} f_{O4}[(A_2+2A_4)d/2k_{11}]$

$$\int_{\theta_0}^{\theta_m} \left(\frac{k_{11}\sin^2\theta + k_{33}\cos^2\theta}{\sin^2\theta_m - \sin^2\theta}\right)^{1/2} d\theta = \frac{d}{2}\frac{\sin\theta_0\cos\theta_0}{[(k_{11}\sin^2\theta_0 + k_{33}\cos^2\theta_0)(\sin^2\theta_m - \sin^2\theta_0)]^{1/2}} \times (A_2 + 2A_4\sin^2\theta_0) \quad \text{for MH},$$

$$\int_{\theta_0}^{\theta_m}\left[\frac{(k_{11}\sin^2\theta + k_{33}\cos^2\theta)(n_e^2\cos^2\theta + n_o^2\sin^2\theta)^{1/2}}{(n_e^2\cos^2\theta + n_o^2\sin^2\theta)^{1/2} - (n_e^2\cos^2\theta_m + n_o^2\sin^2\theta_m)^{1/2}}\right]^{1/2} = \frac{d}{2}\left[\frac{(n_e^2\cos^2\theta_0 + n_o^2\sin^2\theta_0)^{1/2}}{(n_e^2\cos^2\theta_0 + n_o^2\sin^2\theta_0)^{1/2} - (n_e^2\cos^2\theta_m + n_o^2\sin^2\theta_m)^{1/2}}\right]^{1/2} \times \frac{\sin\theta_0\cos\theta_0(A_2 + 2A_4\sin^2\theta_0)}{(k_{11}\sin^2\theta_0 + k_{33}\cos^2\theta_0)^{1/2}} \quad \text{for OH.} \tag{5}$$

The criteria for the first order Fréedericksz transition to occur can be obtained by:

$$\left(\frac{dT}{d\theta_m}\right)_{\theta_m=0} < 0, \quad \left(\frac{dT}{d\theta_m}\right)_{\theta_m=\pi/2} < 0. \tag{6}$$

The first part of equation (6) is for the threshold and the second part for saturation.

All of these criteria are listed in table 1.

3. Remarks on the criteria

$A_4 < 0$ is required for the magnetic and the electrical first order Fréedericksz transition. It is a critical condition, but we have no mature technology to give a minus A_4 surface anchoring condition. A rigid anchoring condition is the most common case, where $A_2 \to \infty$, $A_4 = 0$, and the first order transition cannot occur. This explains why first order transitions in the electrical and magnetic cases are so difficult to observe. The situation is improved when a bias field is applied in addition to the driving field, and observations of first order transitions have been reported [3]. The criteria for the optically first order transition are not so critical, but all of the observations reported so far are made in the presence of bias fields [4]. If we consider a three-dimensional Fréedericksz transition, say in a twist liquid crystal cell, the first order transition can be easily realized when the twist angle $\Phi > 270°$, even for strong anchoring condition [5]. Bistable devices have been developed for $\Phi = 360°$ [6].

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Appendix

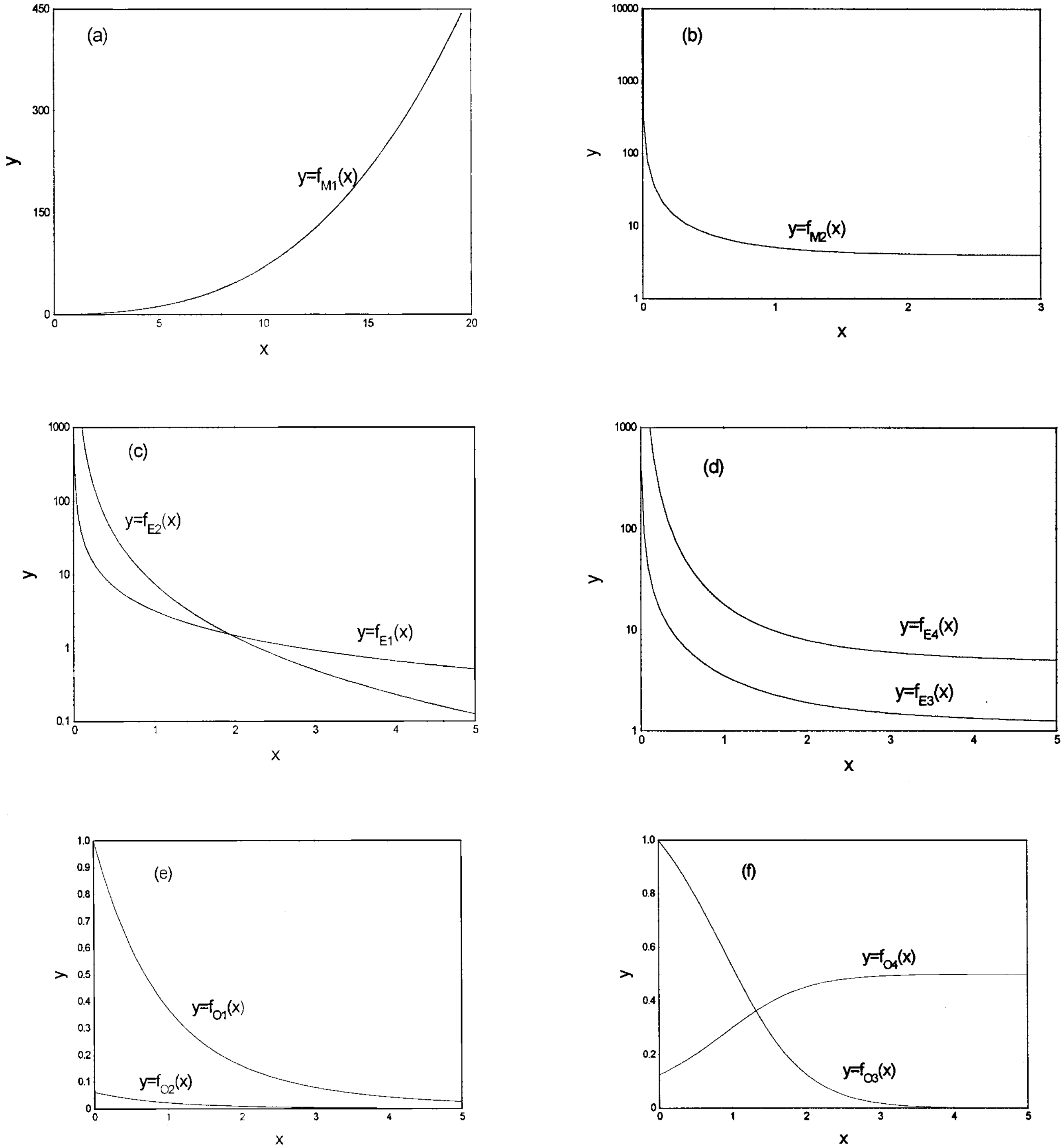

Figure 2. Special functions used in the criteria; all of them are positive. (*a*) and (*b*) are used in the magnetically driven cell; (*c*) and (*d*) are used in the electrically driven cell; (*e*) and (*f*) are used in the optically driven cell.

Table 2. Definitions of special functions used in the criteria; see also figure 2. $\beta \in [0, \pi/2]$, $B \in [0, \infty]$.

x	y	$y = f(x)$
$\beta \tan \beta$	$\frac{16 \sin 2\beta \cos^2 \beta}{4\beta - \sin 4\beta}$	$y = f_{M1}(x)$
$B \tanh B$	$\frac{16 \sinh 2B \cosh^2 B}{\sinh 4B - 4B}$	$y = f_{M2}(x)$
$\beta \tan \beta$	$\frac{2 \sin^2 2\beta - 2\beta \sin 4\beta}{\beta \sin 4\beta + 4\beta^2 - 2 \sin^2 2\beta}$	$y = f_{E1}(x)$
$\beta \tan \beta$	$\frac{16\beta \sin 2\beta \cos^2 \beta}{\beta \sin 4\beta + 4\beta^2 - 2 \sin^2 2\beta}$	$y = f_{E2}(x)$
$B \tanh B$	$\frac{B \sinh 4B - 4B^2}{B \sinh 4B + 4B^2 - 2 \sinh^2 2B}$	$y = f_{E3}(x)$
$B \tanh B$	$\frac{16 B \sinh 2B \cosh^2 B}{B \sinh 4B + 4B^2 - 2 \sinh^2 2B}$	$y = f_{E4}(x)$
$\beta \tan \beta$	$\frac{4 \sin 4\beta + 8 \sin 2\beta}{\sin 4\beta + 8 \sin 2\beta + 12\beta}$	$y = f_{O1}(x)$
$\beta \tan \beta$	$\frac{\sin 2\beta \cos^2 \beta}{\sin 4\beta + 8 \sin 2\beta + 12\beta}$	$y = f_{O2}(x)$
$B \tanh B$	$\frac{4B \operatorname{sech}^4 B + 4 \tanh B \operatorname{sech}^2 B}{3B \operatorname{sech}^4 B + 3 \tanh B \operatorname{sech}^2 B + 2 \tanh B}$	$y = f_{O3}(x)$
$B \tanh B$	$\frac{\tanh B}{3B \operatorname{sech}^4 B + 3 \tanh B \operatorname{sech}^2 B + 2 \tanh B}$	$y = f_{O4}(x)$